

from a point, and the caustic by refraction of a circle for rays proceeding from a point; the result in the last case is not worked out, but it is shown how the equation in rectangular coordinates is to be obtained by equating to zero the discriminant of a rational and integral function of the sixth degree. The memoir treats also of the secondary caustic or orthogonal trajectory of the reflected or refracted rays in the general case of a reflecting or refracting circle and rays proceeding from a point; the curve in question, or rather a secondary caustic, is, as is well known, the Oval of Descartes or 'Cartesian:' the equation is discussed by a method which gives rise to some forms of the curve which appear to have escaped the notice of geometers. By considering the caustic as the evolute of the secondary caustic, it is shown that the caustic in the general case of a reflecting or refracting circle and rays proceeding from a point is a curve of the sixth class only. The concluding part of the memoir treats of the curve which, when the incident rays are parallel, must be taken for the secondary caustic in the place of the Cartesian, which, for the particular case in question, passes off to infinity. In the course of the memoir, the author reproduces a theorem first given, he believes, by himself in the *Philosophical Magazine*, viz. that there are six different systems of a radiant point and refracting circle which give rise to identically the same caustic. The memoir is divided into sections, each of which is to a considerable extent intelligible by itself, and the subject of each section is for the most part explained by the introductory paragraph or paragraphs.

III. "On the Figure, Dimensions, and Mean Specific Gravity of the Earth, as derived from the Ordnance Trigonometrical Survey of Great Britain and Ireland." Communicated by Lieut-Colonel JAMES, R.E., F.R.S., &c., Superintendent of the Ordnance Survey. Received April 30, 1856.

(Abstract.)

The Trigonometrical Survey of the United Kingdom commenced in the year 1784, under the immediate auspices of the Royal Society; the first base was traced by General Roy on the 16th of April of

that year, on Hounslow Heath, in presence of Sir Joseph Banks, then President of the Society, and some of its most distinguished Fellows.

The principal object which the Government had then in view, was the connexion of the Observatories of Paris and Greenwich by means of a triangulation, for the purpose of determining the difference of longitude between the two observatories.

A detailed account of the operations then carried on is given in the first volume of the 'Trigonometrical Survey,' which is a revised account of that which was first published in the 'Philosophical Transactions' for 1785 and three following years.

At the time when these operations were in progress, the Survey of several counties in the south-east of England, including Kent, Sussex, Surrey, and Hampshire, was also in progress, under the direction of the Master-General of the Ordnance, for the purpose of making military maps of the most important parts of the kingdom in a military point of view; and it was then decided to make the triangulation which extended from Hounslow to Dover the basis of a triangulation for these surveys.

It is extremely to be regretted that a more enlarged view of the subject had not then been taken, and a proper geometrical projection made for the map of the whole kingdom. As it is, the south-eastern counties were first drawn and published in reference to the meridian of Greenwich, then Devonshire in reference to the meridian of Butters-ton in that county, and thirdly the northern counties, in reference to the meridian of Delamere in Cheshire; but there is a large intermediate space, the maps of which are made of various sizes to accommodate them to the convergence of the meridian.

In 1799 the Royal Society gave further proof of the interest it took in the progress of the Survey, by lending to the Ordnance its great 3-foot Theodolite, made by Ramsden, for the purpose of expediting the work of the Survey; and although this instrument has been in almost constant use for the last sixty-seven years, during which time it has been placed on the highest church towers and the loftiest mountains in the kingdom, from the Shetlands to the Scilly Islands, it is at this day in perfect working order, and probably one of the very best instruments that was ever made.

The great Trigonometrical operations of the Survey have been

carried on under so many officers, from the time of their commencement under General Roy down to the present time, that it would be quite impossible, in this short notice, to mention more than the names of several Superintendents who have succeeded General Roy, viz. Colonel Williams, Major-General Mudge, Major-General Colby, and Colonel Hall; but in justice to the highly meritorious body of non-commissioned officers of the Corps of Royal Sappers and Miners, it should be stated, that whilst in the early part of the Survey the most important and delicate observations were entrusted solely to the commissioned officers, these duties have of late years been performed by the non-commissioned officers with the greatest skill and accuracy.

The computations connected with the corrections of the observed angles, to make the whole triangulation as nearly as possible perfectly consistent, have been most voluminous, and have been made under the direction of Lieut.-Colonel Yolland, Captain Cameron, and Captain Alexander R. Clarke; but Col. James gladly avails himself of this opportunity to acknowledge the great and important assistance and advice which, both as regards the instruments and the calculations, have at all times been received from the Astronomer Royal.

The triangulation, by the methods which will be explained, is now made consistent in every part, so that any side of any triangle being taken as a base, the same distance will be reproduced when it is computed through any portion or the whole series of triangles; and when the five measured bases relied on are incorporated in this triangulation, the greatest difference between their measured and computed lengths is not as much as 3 inches, and yet some of the bases are upwards of 400 miles apart.

Several bases of from five to seven miles long have been measured, but those upon which the chief reliance has been placed are the Lough Foyle and Salisbury Plain bases which were measured with General Colby's compensation bars. The difference between the measured and computed length of the one base from the other through the triangulation is 0·4178 ft., or about 5 inches.

This difference has been divided in proportion to the square root of the lengths of the measured bases, by which the mean base which has been used in the triangulation has been obtained; there is therefore a difference of + or - 0·2 ft., or $2\frac{1}{2}$ inches between the measured and computed length of these bases from the mean base.

The Hounslow Heath base was measured with Ramsden's 100 ft. steel chains, and only differs 0·173 ft., or about 2 inches, from its computed length from the mean base.

The Belhelvie base in Aberdeenshire, also measured with the steel chains, differs only 0·24 ft., or less than 3 inches, from the computed length.

The difference between the measured and computed length of the Misterton Carr base, near Doncaster, also measured with the steel chains, is only 0·157 ft., or less than 2 inches; and it will be observed that the difference between the computed and measured lengths of these three bases (measured with chains) is not greater than the difference between the measured and computed length of the Lough Foyle and Salisbury Plain bases (measured with the compensation bars), from which it may be inferred, that bases measured with steel chains are deserving of the greatest confidence; and when the great simplicity, portability, and cheapness of the chains are compared with the complex, heavy and expensive apparatus of the compensation bars, it may be anticipated that they will be more generally employed than they have been of late years, especially in the colonies, and in countries where the transport of heavy articles is effected with difficulty.

The length of the base on Rhuddlan Marsh in North Wales, which was measured with steel chains, differs 1·596 ft. from the computed length; but from the circumstance that the extremities of the base are very badly situated with reference to the surrounding Trigonometrical stations, the angles being very acute and not well observed, little confidence has been placed in the result of the comparison of its computed and measured length.

One of the first practical results arising from the completion of the triangulation is, that it is now possible to engrave the latitude and longitude on the marginal lines of the old sheets of the one-inch Map of England, and this is now being done.

The following account of the Trigonometrical operations and calculations has been drawn up by Captain Alexander R. Clarke, R.E.; this account may be considered an abridgement of that more detailed account which is now in the press, and will be shortly published.

It will be seen that the equatorial diameter of the earth, as derived from the Ordnance Survey, is 7926·610 miles, or about one mile

greater than it is given by the Astronomer Royal in his 'Figure of the Earth,' and that the ellipticity is $\frac{1}{299,33}$, or as the Astronomer Royal conjectured, something "greater than $\frac{1}{300}$," which he gives in the same paper.

The mean specific gravity of the earth, as derived from the observations at Arthur's Seat, was stated in a former paper to be 5.14; the calculations have since been revised, and it is now found to be 5.316.

The mean specific gravity of the earth, as derived from the only other observations on the attraction of mountain masses on which any reliance has been placed, viz. the Schehallien observations, is, as finally corrected by Hutton, $\frac{99}{20}$, or almost 5.0.

From the experiments with balls we have the following results:—

By Cavendish, as corrected by Bailly.	5.448
By Bailly	5.67
By Reich.	5.44

From the pendulum experiments, at a great depth and on the surface, the Astronomer Royal obtained 6.566.

Two copies of the new National Standard Yard have recently been received through the Astronomer Royal, and it is obviously necessary that the geodetic measures should be given in reference to the standard; but not knowing from what scale the standard has been taken, Col. James is unable to say at present in what way the reduction is to be made; that is, whether by reference to the comparison of the old standards which have been already made, or by the mechanical process of a direct comparison of the Ordnance Standard with the new National Standard.

This introductory explanation by Col. James is followed by an account of the Trigonometrical operations and calculations; the following is a brief statement of the results:—

"1st. The four bases of verification, when their measured lengths are compared with their lengths as calculated from a mean of the Lough Foyle and Salisbury Plain bases, show the following discrepancies:—

Hounslow.	Misterton Carr.	Rhuddlan Marsh.	Belhelvie.
+0.173	—0.157	+1.596	+0.240

“2nd. The elements of the spheroid most nearly representing the surface of Great Britain are—

$$\begin{array}{rcl} & \text{Feet 0.} & \text{Miles.} \\ \text{Equatorial semidiameter} & = 20926249 = 3963\cdot305 \\ \text{Polar semidiameter} & = 20856337 = 3950\cdot064 \end{array} \left. \vphantom{\begin{array}{rcl} & \text{Feet 0.} & \text{Miles.} \end{array}} \right\} \text{compression} = \frac{1}{299\cdot33}.$$

“3rd. The elements of the spheroid most nearly representing the whole of the measured arcs considered in this paper are—

$$\begin{array}{rcl} & \text{Feet 0.} & \text{Miles.} \\ \text{Equatorial semidiameter} & = 20924969 = 3963\cdot064 \\ \text{Polar semidiameter} & = 20854743 = 3949\cdot760 \end{array} \left. \vphantom{\begin{array}{rcl} & \text{Feet 0.} & \text{Miles.} \end{array}} \right\} \text{compression} = \frac{1}{297\cdot96}.$$

“4th. The lengths of the degrees of latitude and longitude in Great Britain are as in the following table :—

Mean latitude.	From Ordnance Survey.		From the 2nd Spheroid.	
	Length in ft. of 1° of latitude.	Length in ft. of 1° of longitude.	Length in ft. of 1° of latitude.	Length in ft. of 1° of longitude.
50	364936·33	235227·42	364912·65	235215·15
51	364999·14	230312·27	364975·74	230300·33
52	365061·50	225326·39	365038·38	225314·75
53	365123·34	220271·15	365100·51	220259·79
54	365184·58	215148·11	365162·02	215137·12
55	365245·15	209958·83	365222·86	209948·14
56	365304·96	204704·93	365282·94	204694·56
57	365363·96	199387·90	365342·20	199377·84
58	365422·06	194009·37	365400·57	193999·63
59	365479·20	188571·00	365457·97	188561·57
60	365535·30	183074·50	365514·32	183065·41

The Society adjourned to Thursday, May 22nd.